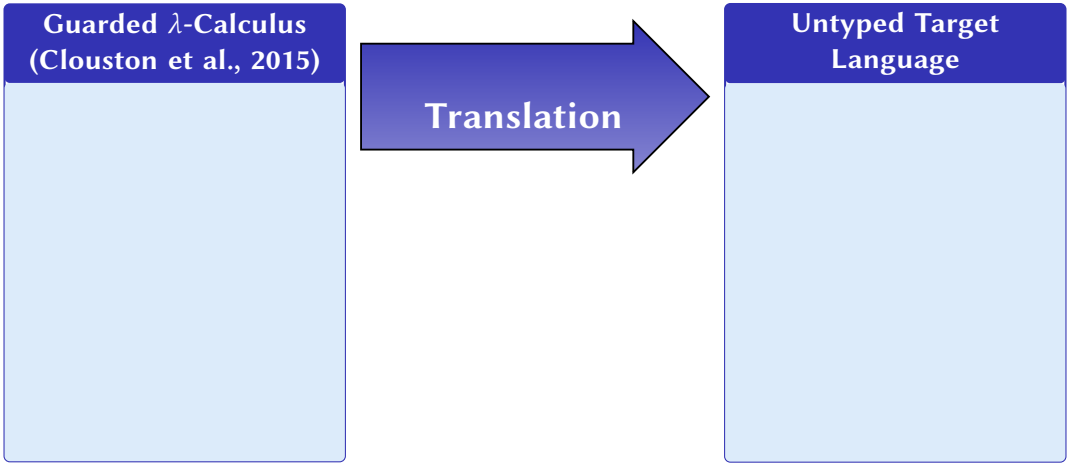
Verified Translation of Guarded Programs

David Läwen, joint work with Robbert Krebbers and Bálint Kocsis

Radboud University

PLNL @ CWI Amsterdam, 28 November 2025

The Main Idea



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Guarded λ-Calculus (Clouston et al., 2015)

- STLC with guarded recursive and coinductive types

Untyped Target Language

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- Resembles e.g. `flambda` intermediate language (OCaml)

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- Mechanised in Rocq using Iris framework

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The Guarded Lambda-Calculus

STLC with Unrestricted Recursive Types

STLC with iso-recursive types $\mu\alpha.\tau$, a.k.a. $\lambda\mu$-calculus

$$\texttt{fold} \;:\; \tau[\mu\alpha.\tau/\alpha] \to \mu\alpha.\tau \qquad\qquad \texttt{unfold} \;:\; \mu\alpha.\tau \to \tau[\mu\alpha.\tau/\tau]$$

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E.g. can express streams over τ

$$\mathrm{Str}\ \tau \;\triangleq\; \mu\alpha.\ \tau \times \alpha \qquad\qquad (::) \;:\; \tau \to \mathrm{Str}\ \tau \to \mathrm{Str}\ \tau$$

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Problem: Type system accepts diverging programs!

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Term can refer to itself only 'later'

$$\mathrm{fix}^{\mathrm{g}}\ :\ (\blacktriangleright\tau \to \tau) \to \tau$$

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Examples:

$$\vdash \mathrm{fix}^{\mathrm{g}}\ (\lambda s : \blacktriangleright\ \mathrm{Str}^{\mathrm{g}}\ \mathbf{N}.\ 0 :: s) : \mathrm{Str}^{\mathrm{g}}\ \mathbf{N}$$

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Problem solved: Diverging programs are rejected.

Guarded Term Formers

New term syntax:

$$\texttt{next} \;:\; \tau \to \blacktriangleright\tau \qquad\qquad \circledast \;:\; \blacktriangleright(\tau_1 \to \tau_2) \to \blacktriangleright\tau_1 \to \blacktriangleright\tau_2$$

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Needed for more interesting examples:

$$\begin{array}{rcll} \text{toggle} & \triangleq & \text{fix}^{\text{g}}\ (\lambda s.\ 1 :: \texttt{next}\ (0 :: s)) & : \quad \text{Str}^{\text{g}}\ \mathbf{N} \\ \text{interleave} & \triangleq & \text{fix}^{\text{g}}\ (\lambda f\ s\ t.\ \text{hd}\ s :: (f \circledast t \circledast \texttt{next}\ (\text{tl}\ s))) & : \quad \text{Str}^{\text{g}}\ \tau \to \blacktriangleright\text{Str}^{\text{g}}\ \tau \to \text{Str}^{\text{g}}\ \tau \end{array}$$

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$$x :: s \triangleq \texttt{fold}\ \langle x, s\rangle$$

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Can we erase `fold`, `unfold`, `next` and $\circledast$ while preserving program behaviour?

Translation & Correctness

Source and Target Language

Goal: Translate $ge \in GExp$ to some $ue \in UExp$

$$
\begin{array}{rcl}
ge & ::= & x \mid \lambda x.ge \mid ge\ ge \\
& \mid & \texttt{lit}\ n \\
& \mid & \langle ge, ge \rangle \mid \langle\rangle \\
& \mid & \texttt{proj}_1\ ge \mid \texttt{proj}_2\ ge \\
& \mid & \texttt{inj}_1\ ge \mid \texttt{inj}_2\ ge \\
& \mid & \texttt{case}\ ge\ \texttt{of}\ x.ge; x.ge \\
& \mid & \texttt{fold}\ ge \mid \texttt{unfold}\ ge \\
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\end{array}
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$$(\!|-|\!) \;:\; GExp \to UExp$$

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$$
\begin{array}{rcl}
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(\!|\texttt{next}\ ge|\!) & \triangleq & \mathit{delay}\ (\!|ge|\!) \\
(\!|ge_1 \circledast ge_2|\!) & \triangleq & \mathit{delay}\ (\mathit{force}\ (\!|ge_1|\!))\ (\mathit{force}\ (\!|ge_2|\!)) \\
\mathit{delay}\ ue & \triangleq & \lambda x.ue,\ x \notin FV(ue) \\
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$$

Correctness of Translation

Theorem (Correctness of $(\!|-|\!)$)

Let $ge \in GExp$. If $\emptyset \vdash ge : \mathbf{N}$, we have that

$$ge \mapsto^* \texttt{lit}\ n \iff (\!|ge|\!) \mapsto^* \texttt{lit}\ n.$$

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Use binary *logical relation* between source and target, indexed by source types

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Novel/interesting aspects

- Reasoning about $\blacktriangleright$ with *abstract* step-indexing

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Novel/interesting aspects

- Reasoning about $\blacktriangleright$ with *abstract* step-indexing
- Relating call-by-name to call-by-value

Outlook & Ongoing Work

- Limitation: Source language cannot express *non-causal* functions, e.g.

$$\text{every2nd}\ (x :: x' :: xs) \quad \triangleq \quad x :: \text{every2nd}\ xs$$

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Future work:

- Extend source with polymorphism and existential types
- Support inductive types alongside guarded recursive types

Bibliography


Clouston, Ranald et al. (2015). ‘Programming and Reasoning with Guarded Recursion for Coinductive Types’. In: *FoSSaCS*. Vol. 9034. LNCS, pp. 407–421. DOI: 10.1007/978-3-662-46678-0_26.


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